Throughput Scaling Laws in Distributed Cognitive Multiple Access Channels

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Abstract

In this paper, we study throughput scaling behaviour of distributed cognitive multiple access channels for two network types: distributed-total-power-interference-limited (DTPIL) networks and interference-limited (DIL) networks.

I. System Model

Consider an underlay cognitive multiple access network as described above: N SUs transmit data to an SBS and interfere with the signal reception at a PBS. Let h_i and g_i represent the fading power gains for the ith STSB and STPB links, respectively. The classical ergodic block fading model [14] is assumed to hold to model variations in channel states for all STPB and STSB links. Further, we assume that h_i's and g_i's are i.i.d. random variables, and the random vectors $\boldsymbol{h} = [h_1, h_2, \ldots, h_N]^\top$ and $\boldsymbol{g} = [g_1, g_2, \ldots, g_N]^\top$ are also independent. We assume that each SU obtain the knowledge of its STSB and STPB channel gains using training signals transmitted by the SBS and the PBS. The communication set-up is represented in Fig. 1 pictorially.

Definition 1.1: We say that the cumulative distribution function (CDF) of a random variable X, denoted by $F(x)$, belongs to the class $\mathcal{C}$-distributions if it satisfies the following properties:

- $F(x)$ is continuous.
- $F(x)$ has a positive support, *i.e.*, $F(x) = 0$ for $x \leq 0$.
- $F(x)$ is strictly increasing, *i.e.*, $F(x_1) < F(x_2)$ for $0 < x_1 < x_2$.
- The tail of $F(x)$ decays to zero *double exponentially*, *i.e.*, there exist constants $\alpha > 0$, $\beta > 0$, $n > 0$, $l \in \mathbb{R}$ and a slowly varying function $H(x)$ satisfying $H(x) = o(x^n)$ such that

$$\lim_{x \to \infty} \frac{1 - F(x)}{\alpha x^l \mathrm{e}^{(-\beta x^n + H(x))}} = 1. \tag{1}$$

- $F(x)$ varies *regularly* around the origin, *i.e.*, there exist constants $\eta > 0$ and $\gamma > 0$ such that

$$\lim_{x \downarrow 0} \frac{F(x)}{\eta x^\gamma} = 1. \tag{2}$$

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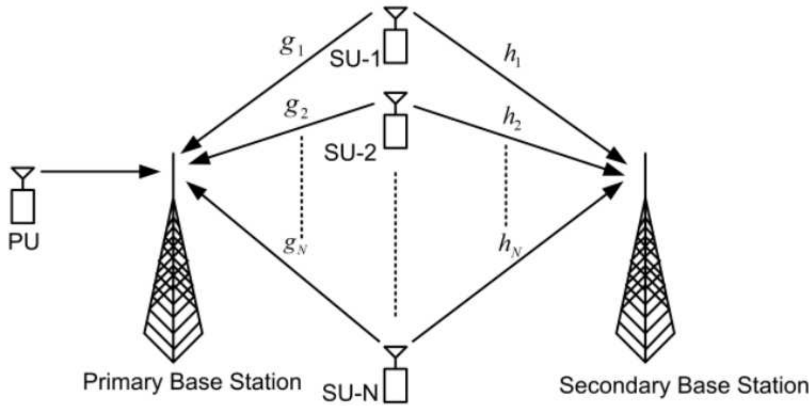

Fig. 1. N SUs forming a multiple access channel to the SBS and interfering with signal reception at the PBS.

TABLE I
COMMON FADING CHANNEL MODELS AND THEIR PARAMETERS

Channel Model	Parameters						
	α	l	β	n	$h(x)$	η	γ
Rayleigh	1	0	1	1	0	1	1
Rician	$\frac{1}{2\sqrt{\pi}\mathrm{e}^{K_f}\sqrt[4]{K_f(K_f+1)}}$	$-\frac{1}{4}$	K_f+1	1	$2\sqrt{K_f\left(K_f+1\right)x}$	$\frac{K_f+1}{\mathrm{e}^{K_f}}$	1
Nakagami-m	$\frac{m^{m-1}}{\Gamma(m)}$	$m-1$	m	1	0	$\frac{m^{m-1}}{\Gamma(m)}$	m
Weibull	1	0	$\Gamma^{\frac{c}{2}}\left(1+\frac{2}{c}\right)$	$\frac{c}{2}$	0	$\Gamma^{\frac{c}{2}}\left(1+\frac{2}{c}\right)$	$\frac{c}{2}$

In this paper, we consider fading power gains with CDFs belonging to the class $\mathcal{C}$-distributions. The parameters characterizing the behavior of the distribution of fading power gains around zero and infinity are illustrated in Table I for the commonly used fading models in the literature. To avoid any confusion, we represent these parameters with subscript h for STSB channel gains and with subscript g for STPB channel gains, *e.g.*, η_g or η_h, in the remainder of the paper.

In this paper, we consider a distributed cognitive multiple access channel with N backlogged SUs where each SU exploits knowledge of its STSB and STPB channel gains to *locally* perform scheduling and power allocation tasks *independent* of other SUs due to lack of a centralized scheduler. We propose threshold based, channel-aware variations of ALOHA protocol which enables each SU to locally carry out its scheduling task. Once a SU decides to transmit, it will employ a water-filling power allocation policy for its transmission. The water-filling power allocation policies are suitably designed to allow SBS to control the total transmission power of secondary network and the average interference power at the PBS. If more than one SU transmits at the same time, SBS declares a collision and the resulting throughput will be zero.

A. DTPIL Networks:

In distributed total power interference limited (DTPIL) networks, transmission powers of SUs are limited by an average total transmission power constraint and a constraint on the average total interference power of SUs at the PBS. In DTPIL networks, transmission power of SU-i, $P_{i,\mathrm{DTPIL}}\left(h_i,g_i\right)$, is given by

$$P_{i,\mathrm{DTPIL}}\left(h_i,g_i\right)=\begin{cases}\left(\frac{1}{\lambda_N+\mu_N g_i}-\frac{1}{h_i}\right)^{+} & \text{if } \frac{h_i}{\lambda_N+\mu_N g_i}>F^{-1}_{\lambda_N,\mu_N}\left(1-p_N\right)\\ 0 & \text{otherwise}\end{cases}, \tag{3}$$

where $F^{-1}_{\lambda_N,\mu_N}\left(x\right)$ is the inverse functional of CDF of $\frac{h_i}{\lambda_N+\mu_N g_i}$, *i.e.*, $F_{\lambda_N,\mu_N}\left(x\right)$, p_N is the identical transmission probability of SUs, λ_N and μ_N are the design parameters adjusted by the secondary network manager (SNM) to control the average total transmission power of the secondary network as well as the average total interference power at the PBS[1]. According to (3), SU-i transmits using water-filling power allocation policy if its joint power and channel state, *i.e.*, $\frac{h_i}{\lambda_N+\mu_N g_i}$ is greater than the threshold value of $F^{-1}_{\lambda_N,\mu_N}\left(1-p_N\right)$. The choice of (3) as power allocation policy in DTPIL networks is motivated by throughput scaling behaviour of centralized total power interference limited (CTPIL) networks under optimal power allocation policy extensively studied in [10] and [11]. In a CTPIL network, STSB and STPB channel gains of all SUs are available at the SBS, and in order to maximize secondary network sum-rate, SBS schedules the SU corresponding to the maximum of $\left\{\frac{h_i}{\lambda_N+\mu_N g_i}\right\}_{i=1}^{N}$. The scheduled SU employs a water-filling power allocation policy with changing power levels [10]. One the other hand, multiuser diversity gain in CTPIL networks depends on the maximum of $\left\{\frac{h_i}{\lambda_N+\mu_N g_i}\right\}_{i=1}^{N}$ which concentrates around $F^{-1}_{\lambda_N,\mu_N}\left(1-\frac{1}{N}\right)$ as the number of SUs become large (see Lemma 2 in [11] for more details). Thus, with $p_N=\frac{1}{N}$, SU-i transmits if it has a high chance of being the SU with the maximum of $\left\{\frac{h_i}{\lambda_N+\mu_N g_i}\right\}_{i=1}^{N}$. Hence, we expect to obtain similar throughput scaling results to that in [10] and [11] for $p_N=\frac{1}{N}$. Later, we show that this choice of transmission probability is asymptotically optimal, *i.e.*, secondary network throughput under $p_N=\frac{1}{N}$ asymptotically is an upper bound for other choices of p_N. Finally, our approach to distributed scheduling problem is different from standard ALOHA protocol in that transmission and scheduling tasks are based on the quality of STSB and STPB channel power gains.

In DTPIL networks, SNM is required to keep the average total transmission power of secondary network and the average total interference power at the PBS below the predetermined levels of P_{ave} and Q_{ave}, respectively. In this paper, we assume that SNM adjusts λ_N and μ_N such that the total transmission power of secondary network and the average interference power at the PBS are satisfied with equality, *i.e.*, we have

$$\mathsf{E}_{\boldsymbol{h},\boldsymbol{g}}\left[\sum_{i=1}^{N}\left(\frac{1}{\lambda_N+\mu_N g_i}-\frac{1}{h_i}\right)^{+}\mathbb{1}_{\left\{\frac{h_i}{\lambda_N+\mu_N g_i}>F^{-1}_{\lambda_N,\mu_N}\left(1-p_N\right)\right\}}\right]=P_{\mathrm{ave}}, \tag{4}$$

[1]We assume that SBS broadcast λ_N and μ_N to all SUs.

and

$$\mathsf{E}_{\boldsymbol{h},\boldsymbol{g}}\left[\sum_{i=1}^{N} g_i\left(\frac{1}{\lambda_N+\mu_N g_i}-\frac{1}{h_i}\right)^{+} 1_{\left\{\frac{h_i}{\lambda_N+\mu_N g_i}>F^{-1}_{\lambda_N,\mu_N}(1-p_N)\right\}}\right]=Q_{\text{ave}}, \tag{5}$$

for all $N\in\mathbb{N}$.

In DTPIL networks, SBS decodes the received signal if just one SU transmits, and if more than one SU transmits at the same time, a collision happens and the resulting throughput will be zero. Hence, the received signal will be successfully decoded if just the SU with the maximum of $\frac{h_i}{\lambda_N+\mu_N g_i}$ transmits. Let $X^{\star}_N\left(\lambda_N,\mu_N\right)$ and $X^{\diamond}_N\left(\lambda_N,\mu_N\right)$ be the largest and the second largest elements among the collection of i.i.d random variables $\left\{X_i\left(\lambda_N,\mu_N\right)\right\}_{i=1}^{N}$, respectively, where $X_i\left(\lambda_N,\mu_N\right)=\frac{h_i}{\lambda_N+\mu_N g_i}$. Then, the sum-rate in DTPIL networks can be expressed as

$$R_{\text{DTPIL}}\left(p_N,N\right)=\mathsf{E}\left[\log\left(X^{\star}_N\left(\lambda_N,\mu_N\right)\right)1_{\{A_N\}}\right] \tag{6}$$

where $A_N=\left\{X^{\star}_N\left(\lambda_N,\mu_N\right)>\max\left(F^{-1}_{\lambda_N,\mu_N}\left(1-p_N\right),1\right),X^{\diamond}_N\left(\lambda_N,\mu_N\right)\leq F^{-1}_{\lambda_N,\mu_N}\left(1-p_N\right)\right\}$.

B. DIL Networks:

In distributed interference limited (DIL) networks, transmission powers of SUs are limited by a constraint on the total average interference power of SUs at the PBS. Similar to DTPIL networks, we consider a threshold based, channel-aware variation of ALOHA protocol where each SU exploits the knowledge of its STSB and STPB channel gains to locally perform the scheduling and power allocation tasks. More specifically, we assume that in DIL network, transmission power of SU-i, $P_{i,\text{DIL}}\left(h_i,g_i\right)$, is given by

$$P_{i,\text{DIL}}\left(h_i,g_i\right)=\begin{cases}\left(\frac{1}{\mu_N g_i}-\frac{1}{h_i}\right)^{+} & \text{if } \frac{h_i}{g_i}>F^{-1}\left(1-p_N\right)\\ 0 & \text{otherwise}\end{cases}, \tag{7}$$

where $F^{-1}(x)$ is the inverse functional of $F(x)$, $F(x)$ is the CDF of $\frac{h_i}{g_i}$, p_N is the transmission probability, identical for all SU, and μ_N is a design parameter adjusted by the SNM to control the total average interference power at the PBS[2]. Based on (7), SU-i transmits using water-filling power allocation policy if its joint power and interference channel state $\frac{h_i}{g_i}$ is greater than the threshold value $F^{-1}\left(1-p_N\right)$. The choice of (7), as power allocation and scheduling policy, is supported by the throughput scaling results obtained in centralized interference limited (CIL) networks under optimal power allocation policy in [10] and [11]. In a CIL network, in order to maximize secondary network sum-rate, SBS schedules the SU with corresponding to the maximum of $\left\{\frac{h_i}{g_i}\right\}_{i=1}^{N}$, and the scheduled SU employs water-filling power allocation policy [10] and [11]. Moreover, multiuser diversity gain in CIL networks depends on the maximum of $\left\{\frac{h_i}{g_i}\right\}_{i=1}^{N}$, and as number of SUs becomes large, maximum of $\left\{\frac{h_i}{g_i}\right\}_{i=1}^{N}$ with high probability takes value around $F^{-1}\left(1-\frac{1}{N}\right)$. Hence, for $p_N=\frac{1}{N}$, the choice of (7), as power allocation and scheduling policy in DIL networks, guarantees that a SU decides to transmit if it has a high

[2]Similar to DTPIL networks, we assume that SNM broadcasts μ_N to all SUs.

chance of being the best SU in the centralized sense. Using (7), we expect to observe similar throughput scaling behavior as that of CIL networks with $p_N = \frac{1}{N}$. Later, we show that this choice of transmission probability is asymptotically optimal, *i.e.*, secondary network throughput under $p_N = \frac{1}{N}$ is asymptotically an upper bound for other choices of p_N.

In DIL networks, SNM is required to keep the average interference power at the PBS below the predetermined value of Q_{ave}. In this paper, we assume that SNM adjusts μ_N such that the average total interfere power at the PBS be equal to Q_{ave}, *i.e.*,

$$\mathsf{E}\left[\sum_{i=1}^{N} g_i \left(\frac{1}{\mu_N g_i} - \frac{1}{h_i}\right)^{+} 1_{\left\{\frac{h_i}{g_i} > F^{-1}(1-p_N)\right\}}\right] = Q_{\text{ave}}, \tag{8}$$

for all $N \in \mathbb{N}$.

Similar to DTPIL networks, in DIL networks, SBS decodes the received signal if just the SU corresponding to the maximum of $\left\{\frac{h_i}{g_i}\right\}_{i=1}^{N}$ transmits, and if more that one SUs transmit at the same time a collision happens and throughput will be zero. let $Y_N^{\star}$ and $Y_N^{\diamond}$ be the largest and the second largest elements of the collection of random variables $\{Y_i\}_{i=1}^{N}$ where $Y_i = \frac{h_i}{g_i}$. Then, the sum-rate in DIL networks can be expressed as

$$R_{\text{DIL}}\left(p_N, N\right) = \mathsf{E}\left[\log\left(\frac{Y_N^{\star}}{\mu_N}\right) 1_{\{B_N\}}\right] \tag{9}$$

where $B_N = \left\{Y_N^{\star} > \max\left(F^{-1}\left(1-p_N\right), \mu_N\right), Y_N^{\diamond} \leq F^{-1}\left(1-p_N\right)\right\}$.

II. Results and Discussions

In this section, we present the main results of the paper along with detailed discussions and numerical studies. All the proofs are relegated to appendices. We start our discussions by establishing secondary network throughput scaling in DTPIL networks for $p_N = \frac{1}{N}$.

Theorem 1: Let $R_{\text{DTPIL}}\left(\frac{1}{N}, N\right)$ be the secondary network sum-rate in DTPIL networks for $p_N = \frac{1}{N}$. Then,

$$\lim_{N\to\infty} \frac{R_{\text{DTPIL}}\left(\frac{1}{N}, N\right)}{\log\log\left(N\right)} = \frac{1}{e n_h}. \tag{10}$$

Proof: Please see Appendix A. ■

Theorem 1 establishes the double logarithmic throughput scaling behavior of secondary network under DTPIL networks when the identical transmission probability of SUs is equal to $\frac{1}{N}$. Theorem 1 also reveals that secondary network throughput in DTPIL networks is affected by a pre-log factor of $\frac{1}{e n_h}$. The result of Theorem 1 has the following intuitive explanation. Note that $\mathsf{Pr}\left(A_N\right)$ represents the fraction of time that only the SU with the maximum of $\frac{h_i}{\lambda_N + \mu_N g_i}$ transmits. In Appendix A, we show that, for $p_N = \frac{1}{N}$, $\mathsf{Pr}\left(A_N\right)$ converges to $\frac{1}{e}$ as N becomes large. Hence, as the number of SUs become large, the fraction of time that just the best SU transmits is approximately equal to $\frac{1}{e}$. Also, in Appendix A, we show that $\log\left(X_N^{\star}\left(\lambda_N, \mu_N\right)\right)$ scales according to $\frac{1}{n_h}\log\log\left(N\right)$. These observations suggest that the secondary network throughput scales according to $\frac{1}{e n_h}\log\log\left(N\right)$ as N becomes

large. Note that these observations just provides an intuitive explanation for the result of Theorem 1, and we provide a rigorous proof for this result in Appendix A.

In Appendix A, we show that the power control parameter λ_N in DTPIL network converges to $\frac{1}{P_{\text{ave}}}$ as N becomes large. We use this fact to establish the logarithmic effect of the average total power constraint, P_{ave}, on the secondary network throughput in DTPIL networks. Based on our analysis in Appendix A, the secondary network sum-rate in DTPIL networks can be written as

$$R_{\text{DTPIL}}\left(\frac{1}{N}, N\right) = \log\left(\frac{1}{\lambda_N}\right) \mathsf{E}\left[1_{\{A_N\}}\right] + \mathsf{E}\left[\log\left(X_N^{\star}\left(1, \frac{\mu_N}{\lambda_N}\right)\right) 1_{\{A_N\}}\right] \tag{11}$$

The first term in (11) converges to $\frac{1}{e}\log\left(P_{\text{ave}}\right)$ as N becomes large which signifies the logarithmic effect of total power constraint on the secondary network throughput in DTPIL networks. Also in Appendix A, it is shown that the second term in (11) scales according to $\frac{1}{en_h}\log\log\left(N\right)$ which shows the double logarithmic effect of number of SUs as well as the effect of fading channel parameters on the secondary network throughput.

Finally, it is insightful to compare throughput scaling behaviour of DTPIL networks with that of CTPIL networks. In [11], it has been shown that the secondary network throughput in CTPIL networks scales according to $\frac{1}{n_h}\log\log\left(N\right)$ when the optimal power allocation policy is employed. This observation suggests that although DTPIL networks do not benefit from a centralized scheduler with full channel state information (FCSI), *i.e.*, STPB and STSB channel gains of all SUs, DTPIL networks are capable of achieving similar throughput scaling to that of a CTPIL network up to a pre-log factor of $\frac{1}{e}$. The pre-log factor of $\frac{1}{e}$ can be interpreted as the price of avoiding feedback and signaling between SUs and SBS which is required in a CTPIL network.

Figure 2 (a)-(d) demonstrates secondary network throughput scaling with the number of SUs in DTPIL network for different communication environments with $p_N = \frac{1}{N}$. In this figure, P_{ave} and Q_{ave} are set to 15dB and 0dB, respectively. Similar qualitative behavior continues to hold for other values of P_{ave} and Q_{ave}. In Fig. 2(a), STSB channel gains are distributed according to the Weibull fading model with $c = 1$ and STPB channel gains are distributed according to the Rayleigh fading model. In Fig. 2(b), STSB channel gains are Weibull distributed with $c = 4$ and STPB channel gains are Rayleigh distributed. As Fig. 2(a) and Fig. 2(b) show, secondary network throughput scales according to $\frac{2}{ec}\log\log\left(N\right)$ with the number of SUs, *i.e.*, $\frac{2}{e}\log\log\left(N\right)$ for $c = 1$ and $\frac{1}{2e}\log\log\left(N\right)$ for $c = 4$, when STSB channel gains are Weibull distributed as predicted by Theorem 1. In Fig 2(c), STSB channel gains are Rayleigh distributed and STPB channel gains are Weibull distributed with $c = 1$ and $c = 2.5$. As Fig. 2(c) shows, secondary network throughput scales according to $\frac{1}{e}\log\log\left(N\right)$ when STSB channel gains are Rayleigh distributed in accordance with Theorem 1. In Fig. 2(c), as the Weibull fading parameter c increases, STPB channel gains become large. Consequently, SUs should reduce their transmission powers to meet the average total interference power constraint at the PBS which results in a throughput reduction. In Fig. 2(d), STSB channel gains are distributed according to the Rayleigh fading model and STPB channel gains are distributed according to the Nakagami-m fading model with $m = 0.5, 1.2$. As Fig. 2(d) shows, secondary network throughput

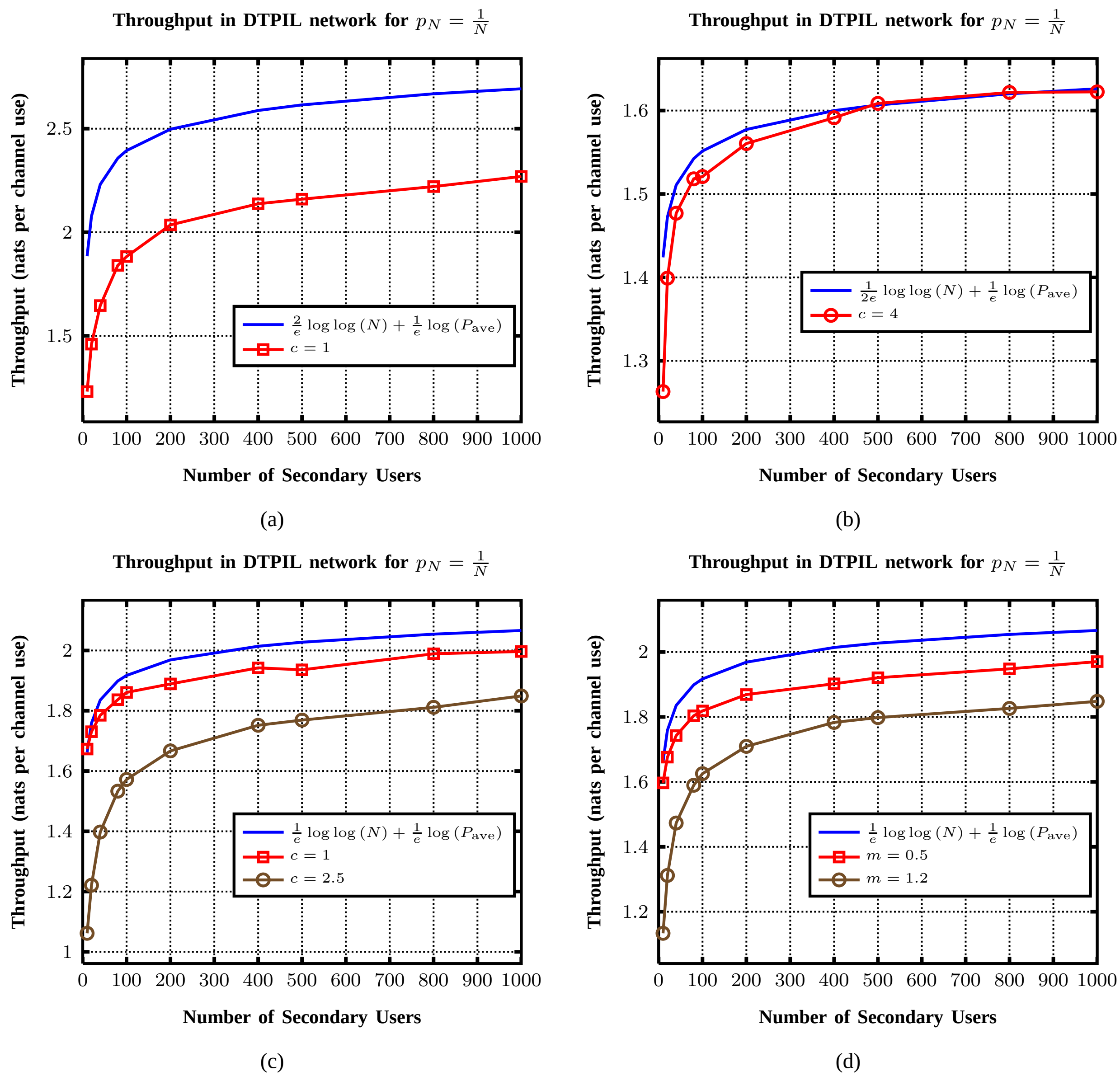

Fig. 2. Secondary network throughput in DTPIL network with number of SUs for different communication environments (a)-(d). $P_{\rm ave}$ and $Q_{\rm ave}$ are set to 15dB and 0dB, respectively.

scales according to $\frac{1}{e}\log\log(N)$ when STSB channel gains are Rayleigh distributed as predicted by Theorem 1. Moreover, as the Nakagmai-m fading parameter m increases in Fig. 2(d), STSB channel gains become more severe, and, as a result SUs reduce their transmission powers to respect the average interference power constraint which results in a throughput drop.

So far, we have assumed that the identical transmission probability, p_N, is equal to $\frac{1}{N}$. Here, we show that this choice of transmission probability is asymptotically optimal, *i.e.*, secondary network throughput scaling under optimal transmission probability will be similar to that of a secondary network under $p_N = \frac{1}{N}$. To this end, we study asymptotic behaviour of optimal transmission probability as number of SUs becomes large.

Theorem 2: For $N \in \mathbb{N}$, let $p_N^\star$ be the optimal transmission probability in DTPIL networks, *i.e.*,

$$p_N^\star = \arg \max_{0 \leq p_N \leq 1} R_{\mathrm{DTPIL}} \left(p_N, N \right).$$

Then, $\lim_{N \to \infty} N p_N^\star = 1$.

Proof: Please see Appendix B. ■

Theorem 2 implies that the sequence of optimal transmission probabilities decays at the rate of $\frac{1}{N}$. This result can be intuitively explained as follows. Assume that $p_N = \frac{a}{N}$ where $0 < a < \infty$. Then, we have $\lim_{N \to \infty} \mathsf{Pr}\left(A_N \right) = \frac{a}{\mathrm{e}^a}$ which implies that the probability of successful transmission[3] is close to $\frac{a}{\mathrm{e}^a}$ as N becomes large. Hence, the secondary network throughput under optimal transmission probability scale according to $\frac{a}{\mathrm{e}^a n_h} \log \log \left(N \right)$. Thus, the best throughput scaling result can be obtained if p_N scales according to $\frac{1}{N}$ which justifies the result of Theorem 2[4]. Moreover, if p_N decays at the rate of $o\left(\frac{1}{N} \right)$ or $\omega\left(\frac{1}{N} \right)$, then, we have $\lim_{N \to \infty} \mathsf{Pr}\left(A_N \right) = 0$ which implies that the probability of successful transmission converges to zero as N becomes large. Hence, the optimal transmission probability can not scale according to $o\left(\frac{1}{N} \right)$ or $\omega\left(\frac{1}{N} \right)$. Based on the result of Theorem 2, we expect that the secondary network throughput under optimal transmission probability scale according to $\frac{1}{\mathrm{e} n_h} \log \log \left(N \right)$. We mention this result formally in the next Theorem.

Theorem 3: Let $R_{\mathrm{DTPIL}} \left(p_N^\star, N \right)$ be the sum-rate in DTPIL networks under $p_N^\star$. Then,

$$\lim_{N \to \infty} \frac{R_{\mathrm{DTPIL}} \left(p_N^\star, N \right)}{\log \log \left(N \right)} = \frac{1}{e n_h}. \tag{12}$$

Proof: Please see Appendix C. ■

Theorem 3 implies that under optimal choice of transmission probability, secondary network throughput scales according to $\frac{1}{\mathrm{e} n_h} \log \log \left(N \right)$. Hence, the choice of transmission probability $p_N = \frac{1}{N}$ is asymptotically optimal, *i.e.*,, one can not obtain better throughput scaling results by other choices of transmission probability. However, it should be noted that Theorem 3 just implies asymptotic optimality of $p_N = \frac{1}{N}$, and, the optimal transmission probability might be different from $\frac{1}{N}$.

Next Theorem establishes throughput scaling behaviour of DIL networks.

Theorem 4: Let $R_{\mathrm{DIL}} \left(p_N, N \right)$ be the secondary network sum-rate in DIL network. Then, for $p_N = \frac{1}{N}$, we have

$$\lim_{N \to \infty} \frac{R_{\mathrm{DIL}} \left(\frac{1}{N}, N \right)}{\log \left(N \right)} = \frac{1}{e \gamma_g}. \tag{13}$$

Proof: Please see Appendix D. ■

Theorem 4 reveals logarithmic behavior of the secondary network throughput under DIL networks. It also implies that the secondary network throughput in DIL networks is affected by a pre-log factor of $\frac{1}{e \gamma_g}$. The result of Theorem 4 can be explained as follows. In DIL networks, $\mathsf{Pr}\left(B_N \right)$ represents the fraction of time that just the best SU

[3]By a successful transmission, we mean just the SU with the maximum of $\frac{h_i}{\lambda_N + \mu_N g_i}$ transmits.

[4] Also, among the class of transmission probabilities decaying at the rate of $\Theta\left(\frac{1}{N} \right)$, asymptotically, the best scaling behavior can be obtained if p_N scales according to $\frac{1}{N}$.

transmits and no collision happens. In Appendix D, we establish that $\lim_{N\to\infty} \mathsf{Pr}\left(B_N\right) = \frac{1}{\mathrm{e}}$ which implies as the number of SUs becomes large, the portion of time that just the best SU transmits is approximately equal to $\frac{1}{e}$. Also, in Appendix D, we show that $\log\left(Y_N^\star\right)$ scales according to $\frac{1}{\gamma_g}\log\left(N\right)$. These observations suggests that secondary network throughput scales according to $\frac{1}{e\gamma_g}\log\left(N\right)$. We give a detailed proof for Theorem 4 in Appendix D.

In Appendix D, we show that the power control parameter in DIL networks, μ_N, converges to $\frac{1}{Q_{\mathrm{ave}}}$ as the number of SUs becomes large. This fact is used to establish logarithmic effect of the average interference power constraint Q_{ave} on the secondary network throughput in DIL networks. Moreover based on our analysis in Appendix D, the secondary network sum-rate in DIL networks under $p_N = \frac{1}{N}$ can be written as

$$R_{\mathrm{DIL}}\left(\frac{1}{N}, N\right) = \log\left(\frac{1}{\mu_N}\right) \mathsf{E}\left[1_{\{B_N\}}\right] + \mathsf{E}\left[\log\left(Y_N^\star\right) 1_{\{B_N\}}\right] \tag{14}$$

The first term in (14) converges to $\frac{1}{e}\log\left(Q_{\mathrm{ave}}\right)$ as N becomes large implying the logarithmic effect of Q_{ave} on the secondary network throughput in DIL networks. We also show that the second term in (14) scales according to $\frac{1}{e\gamma_g}\log\left(N\right)$ signifying the logarithmic effect of number of SUs on the secondary network throughput in DIL networks.

Finally, we compare the results of Theorem 4 with throughput scaling behavior of centralized interference limited (CIL) networks. Secondary network throughput in CIL networks scales according to $\frac{1}{\gamma_g}\log\left(N\right)$ when the optimal power allocation policy is employed [11]. This observation suggests that throughput scaling behavior of DIL networks is similar to that of CIL networks up to a pre-log factor of $\frac{1}{e}$. Similar to DTPIL network, $\frac{1}{e}$ can be interpreted as the cost of implementing a decentralized network with local decision makers.

Fig. 3 (a)-(d) shows secondary network throughput scaling in DIL networks with number of SUs for different communication environments. In this figure, Q_{ave} is set to 0dB. Similar qualitative behavior continue to hold for other value of Q_{ave}. In Fig. 3(a), STSB channel gains are distributed according to Rayleigh fading model and STPB channel gains are distributed according to Weibull fading model with $c = 1$. In Fig. 3(b), STSB channel gains are distributed according to Rayleigh fading model and STPB channel gains are distributed according to Weibull fading model with $c = 4$. As Fig. 3(a) and Fig. 3(b) show, secondary network throughput scales according to $\frac{2}{ec}\log\left(N\right)$ with number of SUs, *i.e.*, $\frac{2}{e}\log\left(N\right)$ for $c = 1$ and $\frac{1}{2e}\log\left(N\right)$ for $c = 4$, when STPB channel gains are Weibull distributed in accordance with Theorem 4. In Fig. 3(c), STSB channel gains are Rayleigh distributed and STPB channel gains are Nakagami-m distributed with $m = 0.5$. In Fig. 3(d), STSB channel gains are Rayleigh distributed and STPB channel gains are Nakagami-m distributed with $m = 1.2$. As Fig. 3(c) and Fig. 3(d) show, secondary network throughput scales according to $\frac{1}{e\gamma}\log\left(N\right)$ with number of SUs, *i.e.*, $\frac{2}{e}\log\left(N\right)$ for $m = 0.5$ and $\frac{1}{1.2e}\log\left(N\right)$ for $m = 1.2$, in DIL networks when STPB channel gains are Nakagami-m distributed as predicted by Theorem 4.

In the next Lemma, we study asymptotic behaviour of sequences of optimal transmission probabilities.

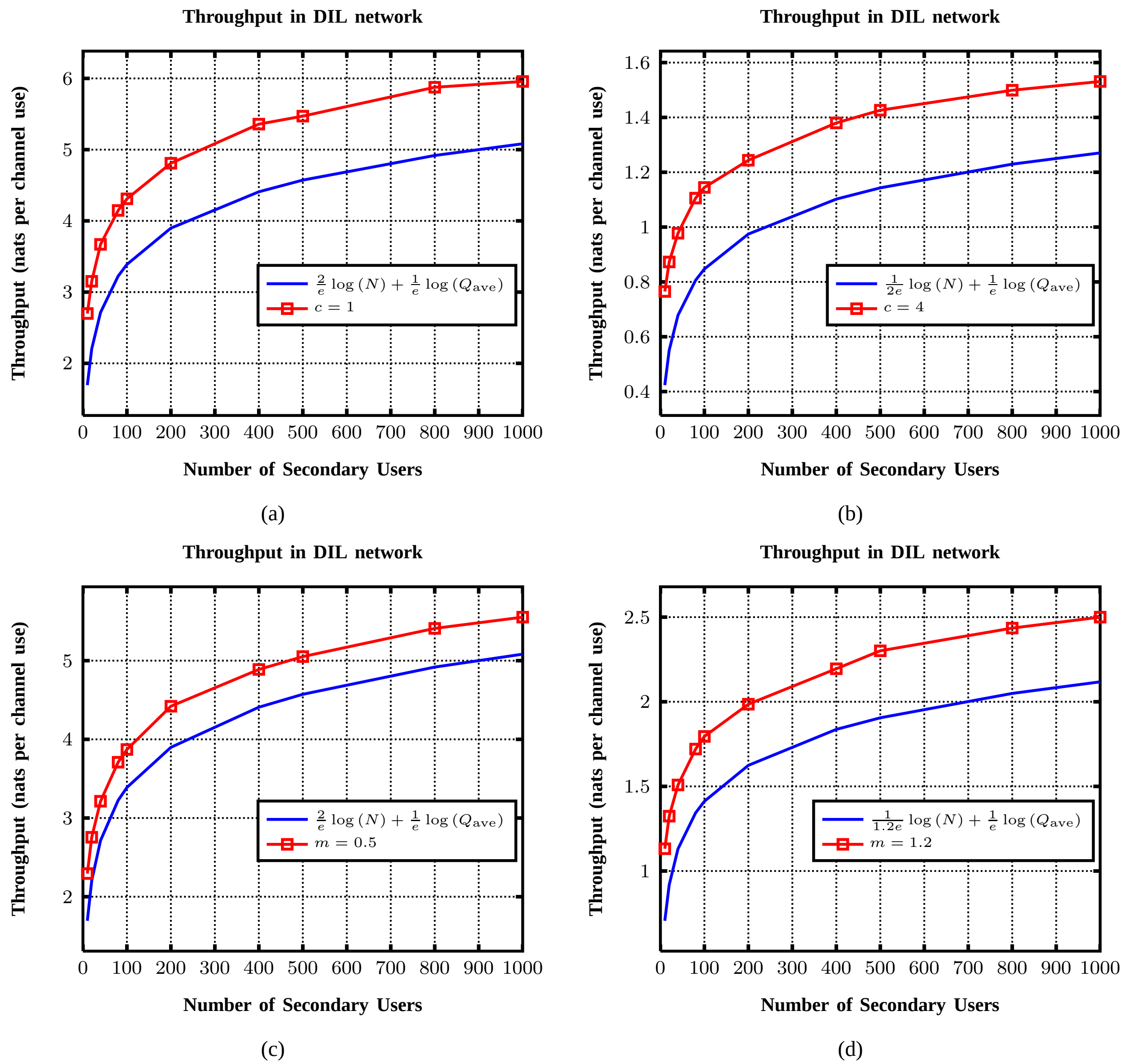

Fig. 3. Secondary network throughput in DIL network with number of SU for different communication environments (a)-(d). Q_{ave} is set to 0dB.

Theorem 5: For $N \in \mathbb{N}$, let $p_N^\star$ be the optimal transmission probability in DIL networks, *i.e.*,

$$p_N^\star = \arg \max_{0 \le p_N \le 1} R_{\text{DTPIL}}\left(p_N, N\right).$$

Then, $\lim_{N\to\infty} N p_N^\star = 1$.

Proof: Please see Appendix E. ■

Theorem 5 implies that in DIL networks, the sequence of optimal transmission probabilities scales according to $\frac{1}{N}$. This result can intuitively be explained as follows. Consider a sequence of transmission probability $\{p_N\}_{N=1}^{\infty}$. Under $p_N = \frac{a}{N}$ with $0 < a < \infty$, we have $\lim_{N\to\infty} \mathsf{Pr}\left(B_N\right) = \frac{a}{\mathrm{e}^a}$ which suggests that the fraction of time that just the SU with the maximum of $\frac{h_i}{g_i}$ transmits is close to $\frac{a}{\mathrm{e}^a}$ as N becomes large. This observation implies that among the class of sequences of transmission probabilities which decay according to $\Theta\left(\frac{1}{N}\right)$, the best scaling result is

obtained by $p_N = \frac{1}{N}$. On the other hand, $p_N^\star$ can not decay faster or slower than $\Theta\left(\frac{1}{N}\right)$. If p_N decays at a rate faster than $\Theta\left(\frac{1}{N}\right)$, *i.e.*, $p_N = o\left(\frac{1}{N}\right)$, we have $\lim_{N\to\infty} \mathsf{Pr}\left(B_N\right) = 0$ which implies that secondary network throughput converges to zero as N becomes large[5]. Similarly, if p_N decays slower than $\Theta\left(\frac{1}{N}\right)$, *i.e.*, $p_N = \omega\left(\frac{1}{N}\right)$, we have $\lim_{N\to\infty} \mathsf{Pr}\left(B_N\right) = 0$. This also implies that secondary network throughput converges to zero as N becomes large[6]. These observations suggests that the sequence of optimal transmission probabilities decays according to $\frac{1}{N}$. Using Theorem 5 and following similar steps to the proof of Theorem 4, we can characterize throughout scaling behaviour of secondary network under $p_N^\star$. We formally mention this result in the next Theorem.

Theorem 6: Let $R_{\text{DIL}}\left(p_N^\star, N\right)$ be the sum-rate in DTPIL networks under $p_N^\star$. Then,

$$\lim_{N\to\infty} \frac{R_{\text{DIL}}\left(p_N^\star, N\right)}{\log\left(N\right)} = \frac{1}{e\gamma_g}. \tag{15}$$

Proof: The proof is similar to that of Theorem 3 and is skipped to avoid repetition. ■

Theorem 6 establishes the logarithmic throughput scaling behaviour of DIL under optimal transmission probability. Hence, the choice of $p_N = \frac{1}{N}$ is asymptotically optimal, and one can not obtain better throughput scaling results by other choices of p_N.

III. CONCLUSION

APPENDIX A
THROUGHPUT SCALING IN DTPIL NETWORKS

In this Appendix, first, we establish some preliminary results. Later, we use these results to prove Theorem 1. In the next Lemma, we study asymptotic behavior of $F_{\lambda,\mu}^{-1}\left(x\right)$ as x becomes close to one. Lemma 1 will be used to study asymptotic behavior of λ_N as N becomes large.

Lemma 1: Let $F_{\lambda,\mu}\left(x\right)$ be the CDF of $\frac{h_i}{\lambda+\mu g_i}$ where λ and μ are positive constants. Then,

$$\lim_{x\uparrow 1} \frac{F_{\lambda,\mu}^{-1}\left(x\right)}{\frac{1}{\lambda}\left(-\frac{1}{\beta_h}\log\left(1-x\right)\right)^{\frac{1}{n_h}}} = 1. \tag{16}$$

Proof: First, we study the tail asymptotic behavior of $F_{\lambda,\mu}\left(x\right)$ as x be comes large. Then, we use this result to obtain behavior of $F_{\lambda,\mu}^{-1}\left(x\right)$ around one. Tail asymptotic behavior of product of two independent random variables has been studied in [22] for the case that $H\left(x\right)$ is equal to zero. Since in our set up $H\left(x\right)$ is not equal to zero, we need to do some work in order to apply the result of [22]. For $\epsilon > 0$, let $h_{+\epsilon}$ and $h_{-\epsilon}$ be two random variables, independent of g_i, with CDFs $F_{+\epsilon}(x)$ and $F_{-\epsilon}(x)$, respectively where tails asymptotic behavior of $F_{+\epsilon}(x)$ and

[5] It is easy to see that if $p_N = o\left(\frac{1}{N}\right)$, no SU transmits as N becomes large since $\frac{F^{-1}_{\lambda_N,\mu_N}\left(1-p_N\right)}{F^{-1}_{\lambda_N,\mu_N}\left(1-\frac{1}{N}\right)}$ grows to infinity as N grows to infinity. Recall that maximum of $\frac{h_i}{\lambda_N+\mu_N g_i}$ concentrates around $F^{-1}_{\lambda_N,\mu_N}\left(1-\frac{1}{N}\right)$.

[6] It is easy to see that for $p_N = \omega\left(\frac{1}{N}\right)$, the fraction of time that a collision happens converges to one.

$F_{-\epsilon}(x)$ are given by:

$$\lim_{x\to\infty}\frac{1-F_{+\epsilon}(x)}{\alpha_h x^{l_h}\mathrm{e}^{\left(-(\beta_h-\epsilon)x^{n_h}\right)}}=\lim_{x\to\infty}\frac{1-F_{-\epsilon}(x)}{\alpha_h x^{l_h}\mathrm{e}^{\left(-(\beta_h+\epsilon)x^{n_h}\right)}}=1.$$

Hence for x large enough, we have $F_{+\epsilon}(x)\le F_{h_i}(x)\le F_{-\epsilon}(x)$ where $F_{h_i}(x)$ is the CDF of h_i. Note that $F_{\lambda,\mu}(x)$ can be written as $F_{\lambda,\mu}(x)=\mathsf{E}_{g_i}\left[F_{h_i}\left(\left(\lambda+\mu g_i\right)x\right)\right]$. Since g_i is a positive random variable and λ and μ are positive constants, $F_{\lambda,\mu}(x)$ can be upper and lower bounded as

$$\mathsf{E}_{g_i}\left[F_{+\epsilon}\left(\left(\lambda+\mu g_i\right)x\right)\right]\le F_{\lambda,\mu}(x)\le \mathsf{E}_{g_i}\left[F_{-\epsilon}\left(\left(\lambda+\mu g_i\right)x\right)\right],$$

for x large enough. Let $F_{+\epsilon,\lambda,\mu}\left(x\right)$ and $F_{-\epsilon,\lambda,\mu}\left(x\right)$ be the CDFs of $\frac{h_{+\epsilon}}{\lambda+\mu g_i}$ and $\frac{h_{-\epsilon}}{\lambda+\mu g_i}$, respectively. Hence, we have $F_{+\epsilon,\lambda,\mu}\left(x\right)\le F_{\lambda,\mu}(x)\le F_{-\epsilon,\lambda,\mu}\left(x\right)$ for x large enough.

Here, we use the results of [22] to characterize tail asymptotic behaviour of $F_{+\epsilon,\lambda,\mu}\left(x\right)$ and $F_{-\epsilon,\lambda,\mu}\left(x\right)$. Let V and U be two positive independent random variables with $\operatorname{esssup} V=\infty$ and $\operatorname{esssup} U=\sigma$ where tail asymptotic behavior of V and U are given by $\lim_{x\to\infty}\frac{1-F_V(x)}{C_V x^t \mathrm{e}^{\left(-K_V x^r\right)}}=1$ and $\lim_{x\uparrow\sigma}\frac{1-F_U(x)}{C_U(\sigma-x)^w}=1$, respectively, for some $C_V>0$, $C_U>0$, $K_V>0$, $r>0$, $w>0$ and $t\in\mathbb{R}$. Then, tail asymptotic behavior of UV is given by $\lim_{x\to\infty}\frac{1-F_{UV}(x)}{C_{UV}x^{t-rw}\mathrm{e}^{\left(-K_V\left(\frac{x}{\sigma}\right)^r\right)}}=1$ where $C_{UV}=C_U C_V\Gamma\left(w+1\right)\sigma^{rw+w-t}\left(K_V r\right)^{-w}$ (see Theorem 3 in [22]). We use this result to characterize the tail asymptotic behavior of $F_{+\epsilon,\lambda,\mu}\left(x\right)$. In this case, $\operatorname{esssup} h_{+\epsilon}=\infty$ and $\operatorname{esssup}\frac{1}{\lambda+\mu g}=\frac{1}{\lambda}$. Also, it is easy to show that the tail behavior of $\frac{1}{\lambda+\mu g}$ is given by $\lim_{x\uparrow\frac{1}{\lambda}}\frac{1-F_{\frac{1}{\lambda+\mu g}}(x)}{\eta_g\left(\frac{\lambda^2}{\mu}\right)^{\gamma_g}\left(\frac{1}{\lambda}-x\right)^{\gamma_g}}=1$ where $F_{\frac{1}{\lambda+\mu g}}\left(x\right)$ is the CDF of $\frac{1}{\lambda+\mu g}$. Hence, the tail asymptotic behavior of $F_{+\epsilon,\lambda,\mu}\left(x\right)$ is given by $\lim_{x\to\infty}\frac{1-F_{+\epsilon,\lambda,\mu}(x)}{Cx^{l_h-n_h\gamma_g}\mathrm{e}^{-\left(\beta_h-\epsilon\right)\left(\lambda x\right)^{n_h}}}=1$ where $C=\eta_g\alpha_h\Gamma\left(\gamma_g+1\right)\left(\frac{\lambda^2}{\mu(\beta_h-\epsilon)n_h}\right)^{\gamma_g}\left(\frac{1}{\lambda}\right)^{n_h\gamma_g+\gamma_g-l_h}$ and $\Gamma\left(\cdot\right)$ is the Gamma function. Using tail asymptotic behavior of $F_{+\epsilon,\lambda,\mu}\left(x\right)$, the behavior of $F^{-1}_{+\epsilon,\lambda,\mu}\left(x\right)$ around one can be characterized as $\lim_{x\uparrow 1}\frac{F^{-1}_{+\epsilon,\lambda,\mu}(x)}{\frac{1}{\lambda}\left(-\frac{1}{(\beta_h-\epsilon)}\log(1-x)\right)^{\frac{1}{n_h}}}=1$. Similarly, we have $\lim_{x\uparrow 1}\frac{F^{-1}_{-\epsilon,\lambda,\mu}(x)}{\frac{1}{\lambda}\left(-\frac{1}{(\beta_h+\epsilon)}\log(1-x)\right)^{\frac{1}{n_h}}}=1$. Note that for x close enough to one and for all $\epsilon>0$, we have $F^{-1}_{-\epsilon,\lambda,\mu}\left(x\right)\le F^{-1}_{\lambda,\mu}(x)\le F^{-1}_{+\epsilon,\lambda,\mu}\left(x\right)$ which implies

$$\lim_{x\uparrow 1}\frac{F^{-1}_{\lambda,\mu}\left(x\right)}{\frac{1}{\lambda}\left(-\frac{1}{\beta_h}\log\left(1-x\right)\right)^{\frac{1}{n_h}}}=1. \tag{17}$$

■

In the next Lemma, we study asymptotic behavior of extreme order statistic of collection of random variables $\left\{\frac{h_i}{\lambda+\mu g_i}\right\}_{i=1}^{N}$. The result of this Lemma will be helpful in studying asymptotic behavior of λ_N as well as proving the main result of Theorem 1.

Lemma 2: Let $X^\star_N\left(\lambda,\mu\right)=\max_{1\le i\le N}\frac{h_i}{\lambda+\mu g_i}$. Then, $\frac{X^\star_N(\lambda,\mu)}{\left(\frac{1}{\beta_h}\log(N)\right)^{\frac{1}{n_h}}}\xrightarrow{i.p.}\frac{1}{\lambda}$ where $i.p.$ stands for convergence in probability.

Proof: Let $F_{\lambda,\mu}\left(x\right)$ be the CDF of $\frac{h_i}{\lambda+\mu g_i}$. Based on Lemma 2 in [11], concentration behavior of $X^\star_N\left(\lambda,\mu\right)$ is characterized by $G^{-1}(x)$ which $G(x)$ is given by $\lim_{x\to\infty}G(x)\left(1-F_{\lambda,\mu}\left(x\right)\right)=1$. Using Lemma 1, it is easy

to show that $G^{-1}(x)$ scales according to $\frac{1}{\lambda}\left(\frac{1}{\beta_h}\log(x)\right)^{\frac{1}{n_h}}$. The desired result follows from Lemma 2 in [11]. ■

Next Lemma establishes an important behavior of λ_N as N becomes large. This Lemma will be used to study the effect of the average total power constraint P_{ave} on the secondary network throughput in DTPIL networks.

Lemma 3: Let λ_N be the power control parameter in DTPIL network. Then, $\lim_{N\to\infty}\lambda_N = \frac{1}{P_{\text{ave}}}$.

Proof: First, we show that λ_N can not be arbitrarily close to zero as N becomes large, ***i.e.***, $\liminf_{N\to\infty}\lambda_N > 0$. Then, we use this result to prove this Lemma. Assume that $\liminf_{N\to\infty}\lambda_N = 0$. Then, for $\epsilon > 0$, we can find a subsequence of N, N_j, such that $\lambda_{N_j} \leq \epsilon$. Let $P_N^{\text{DTPIL}}(\boldsymbol{h},\boldsymbol{g}) = \sum_{i=1}^{N}\left(\frac{1}{\lambda_N+\mu_N g_i} - \frac{1}{h_i}\right)^{+}\mathsf{1}_{\left\{\frac{h_i}{\lambda_N+\mu_N g_i} > F^{-1}_{\lambda_N,\mu_N}\left(1-\frac{1}{N}\right)\right\}}$ be the instantaneous total power in DTPIL networks. Then, average power constraint can be lower bounded as

$$\begin{aligned}
\mathsf{E}\left[P_{N_j}^{\text{DTPIL}}(\boldsymbol{h},\boldsymbol{g})\right] &= \mathsf{E}\left[\sum_{i=1}^{N_j}\left(\frac{1}{\lambda_{N_j}+\mu_{N_j}g_i} - \frac{1}{h_i}\right)^{+}\mathsf{1}_{\left\{\frac{h_i}{\lambda_{N_j}+\mu_{N_j}g_i} > F^{-1}_{\lambda_{N_j},\mu_{N_j}}\left(1-\frac{1}{N_j}\right)\right\}}\right] \\
&\geq \mathsf{E}\left[\left(\frac{1}{\lambda_{N_j}+\mu_{N_j}g_{I_j}} - \frac{1}{h_{I_j}}\right)^{+}\mathsf{1}_{\left\{X^{\star}_{N_j}\left(\lambda_{N_j},\mu_{N_j}\right) > F^{-1}_{\lambda_{N_j},\mu_{N_j}}\left(1-\frac{1}{N_j}\right)\right\}}\right] \\
&= \mathsf{E}\left[\frac{1}{h_{I_j}}\left(X^{\star}_{N_j}\left(\lambda_{N_j},\mu_{N_j}\right) - 1\right)^{+}\mathsf{1}_{\left\{X^{\star}_{N_j}\left(\lambda_{N_j},\mu_{N_j}\right) > F^{-1}_{\lambda_{N_j},\mu_{N_j}}\left(1-\frac{1}{N_j}\right)\right\}}\right] \\
&\overset{(a)}{\geq} \mathsf{E}\left[\frac{1}{h^{\star}_{N_j}}\left(X^{\star}_{N_j}\left(\epsilon,\frac{1}{Q_{\text{ave}}}\right) - 1\right)^{+}\mathsf{1}_{\left\{X^{\star}_{N_j}\left(\lambda_{N_j},\mu_{N_j}\right) > F^{-1}_{\lambda_{N_j},\mu_{N_j}}\left(1-\frac{1}{N_j}\right)\right\}}\right], \qquad (18)
\end{aligned}$$

where $X^{\star}_{N_j}\left(\epsilon,\frac{1}{Q_{\text{ave}}}\right) = \max_{1\leq i\leq N_j}\frac{h_i}{\epsilon+\frac{1}{Q_{\text{ave}}}g_i}$, $h^{\star}_{N_j} = \max_{1\leq i\leq N_j} h_i$ and $I_j = \arg\max_{1\leq i\leq N_j}\frac{h_i}{\lambda_{N_j}+\mu_{N_j}g_i}$, and (a) follows form the fact hat $\mu_N \leq \frac{1}{Q_{\text{ave}}}$. Using Lemma 2, we have $\frac{X^{\star}_{N_j}\left(\epsilon,\frac{1}{Q_{\text{ave}}}\right)}{\left(\frac{1}{\beta_h}\log(N_j)\right)^{\frac{1}{n_h}}} \xrightarrow{i.p.} \frac{1}{\epsilon}$ and $\frac{h^{\star}_{N_j}}{\left(\frac{1}{\beta_h}\log(N_j)\right)^{\frac{1}{n_h}}} \xrightarrow{i.p.} 1$. Also, it is easy to show that

$$\mathsf{1}_{\left\{X^{\star}_{N_j}\left(\lambda_{N_j},\mu_{N_j}\right) > F^{-1}_{\lambda_{N_j},\mu_{N_j}}\left(1-\frac{1}{N_j}\right)\right\}} \xrightarrow{i.d.} Bern\left(1-\frac{1}{e}\right), \qquad (19)$$

where $Bern(p)$ denotes a 0-1 Bernoulli random variable with mean p, and $i.d.$ stands for in distribution. Using Slutsky Theorem [21], we have $\frac{1}{h^{\star}_{N_j}}\left(X^{\star}_{N_j}\left(\epsilon,\frac{1}{Q_{\text{ave}}}\right) - 1\right)^{+}\mathsf{1}_{\left\{X^{\star}_{N_j}\left(\lambda_{N_j},\mu_{N_j}\right) > F^{-1}_{\lambda_{N_j},\mu_{N_j}}\left(1-\frac{1}{N_j}\right)\right\}} \xrightarrow{i.d.} \frac{1}{\epsilon}Bern\left(1-\frac{1}{e}\right)$. Applying Fatu's Lemma to (18), we have $\liminf_{N\to\infty}\mathsf{E}\left[P_{N_j}^{\text{TPIL}}(\boldsymbol{h},\boldsymbol{g})\right] \geq \frac{1}{\epsilon}\left(1-\frac{1}{e}\right)$. This implies that the average total power can arbitrarily large for ϵ small enough and N_j large enough. Thus, $\liminf_{N\to\infty}\lambda_N > 0$.

Now, we show that $\lim_{N\to\infty}\lambda_N = \frac{1}{P_{\text{ave}}}$. We assume that the average total power constraint is satisfied with equality for all N. Note that $\lambda_N \leq \frac{1}{P_{\text{ave}}}$ which implies that $\limsup_{N\to\infty}\lambda_N \leq \frac{1}{P_{\text{ave}}}$. The average total power can

be lower bounded as

$$\begin{aligned} P_{\text{ave}} &= \mathsf{E}\left[\sum_{i=1}^{N}\left(\frac{1}{\lambda_N+\mu_N g_i}-\frac{1}{h_i}\right)^{+}1_{\left\{\frac{h_i}{\lambda_N+\mu_N g_i}>F^{-1}_{\lambda_N,\mu_N}\left(1-\frac{1}{N}\right)\right\}}\right] \\ &\overset{(a)}{=} \frac{1}{\lambda_N}\mathsf{E}\left[\sum_{i=1}^{N}\left(\frac{1}{1+\frac{\mu_N}{\lambda_N}g_i}-\frac{\lambda_N}{h_i}\right)^{+}1_{\left\{\frac{h_i}{1+\frac{\mu_N}{\lambda_N}g_i}>F^{-1}_{1,\frac{\mu_N}{\lambda_N}}\left(1-\frac{1}{N}\right)\right\}}\right] \\ &\geq \frac{1}{\lambda_N}\mathsf{E}\left[\sum_{i=1}^{N}\frac{1}{h_i}\left(F^{-1}_{1,\frac{1}{\lambda_N Q_{\text{ave}}}}\left(1-\frac{1}{N}\right)-\lambda_N\right)^{+}1_{\left\{\frac{h_i}{1+\frac{\mu_N}{\lambda_N}g_i}>F^{-1}_{1,\frac{\mu_N}{\lambda_N}}\left(1-\frac{1}{N}\right)\right\}}\right] \\ &\geq \frac{1}{\lambda_N}\mathsf{E}\left[\frac{1}{h_N^{\star}}\left(F^{-1}_{1,\frac{1}{\lambda_N Q_{\text{ave}}}}\left(1-\frac{1}{N}\right)-\lambda_N\right)^{+}\sum_{i=1}^{N}1_{\left\{\frac{h_i}{1+\frac{\mu_N}{\lambda_N}g_i}>F^{-1}_{1,\frac{\mu_N}{\lambda_N}}\left(1-\frac{1}{N}\right)\right\}}\right], \end{aligned} \tag{20}$$

where (a) follows from the fact that $\lambda F^{-1}_{\lambda,\mu}(x) = F^{-1}_{1,\frac{\mu}{\lambda}}(x)$. Using (20), λ_N can be lower bounded as

$$\lambda_N \geq \frac{1}{P_{\text{ave}}}\mathsf{E}\left[\frac{1}{h_N^{\star}}\left(F^{-1}_{1,\frac{1}{\lambda_N Q_{\text{ave}}}}\left(1-\frac{1}{N}\right)-\lambda_N\right)^{+}\sum_{i=1}^{N}1_{\left\{\frac{h_i}{1+\frac{\mu_N}{\lambda_N}g_i}>F^{-1}_{1,\frac{\mu_N}{\lambda_N}}\left(1-\frac{1}{N}\right)\right\}}\right]. \tag{21}$$

Using Lemma 1, we have $\lim_{N\to\infty}\frac{F^{-1}_{1,\frac{1}{\lambda_N Q_{\text{ave}}}}\left(1-\frac{1}{N}\right)}{\left(\frac{1}{\beta_h}\log(N)\right)^{\frac{1}{n_h}}} = 1$ since λ_N can not be arbitrarily close to zero. Thus, $\frac{\left(F^{-1}_{1,\frac{1}{\lambda_N Q_{\text{ave}}}}\left(1-\frac{1}{N}\right)-\lambda_N\right)^{+}}{h_N^{\star}} \xrightarrow{i.p.} 1$. Let $S_N = \sum_{i=1}^{N}1_{\left\{\frac{h_i}{1+\frac{\mu_N}{\lambda_N}g_i}>F^{-1}_{1,\frac{\mu_N}{\lambda_N}}\left(1-\frac{1}{N}\right)\right\}}$. For $k \in \mathbb{N}$ and $x \in [k, k+1)$, $\mathsf{Pr}\left(S_N \leq x\right)$ is given by

$$\mathsf{Pr}\left(S_N \leq x\right) = \begin{cases} \binom{N}{k}\left(\frac{1}{N}\right)^{k}\left(1-\frac{1}{N}\right)^{N-k} & \text{if } k \leq N \\ 1 & N < k \end{cases}. \tag{22}$$

Hence, $\lim_{N\to\infty}\mathsf{Pr}\left(S_N \leq x\right) = \frac{1}{k!}e^{-1}$ which implies that S_N converges in distribution to $Po\left(1\right)$ where $Po\left(p\right)$ represents a Poisson random variable with mean p. Using Slutsky Theorem, we have

$$\frac{1}{h_N^{\star}}\left(F^{-1}_{1,\frac{1}{\lambda_N Q_{\text{ave}}}}\left(1-\frac{1}{N}\right)-\lambda_N\right)^{+}\sum_{i=1}^{N}1_{\left\{\frac{h_i}{1+\frac{\mu_N}{\lambda_N}g_i}>F^{-1}_{1,\frac{\mu_N}{\lambda_N}}\left(1-\frac{1}{N}\right)\right\}} \xrightarrow{i.d.} Po\left(1\right). \tag{23}$$

Applying Fatou's Lemma to (21), we have $\liminf_{N\to\infty}\lambda_N \geq \frac{1}{P_{\text{ave}}}$ which completes the proof. ■

Now, we are ready to establish the result of Theorem 1. Note that the sum-rate in DTPIL networks can be written as

$$R_{\text{DTPIL}}\left(\frac{1}{N}, N\right) = \log\left(\frac{1}{\lambda_N}\right)\mathsf{E}\left[1_{\{A_N\}}\right] + \mathsf{E}\left[\log\left(X_N^{\star}\left(1, \frac{\mu_N}{\lambda_N}\right)\right)1_{\{A_N\}}\right]. \tag{24}$$

For N large enough, $1_{\{A_N\}}$ can be written as

$$1_{\{A_N\}} = \sum_{i=1}^{N} 1_{\left\{\frac{h_i}{\lambda_N+\mu_N g_i} > F^{-1}_{\lambda_N,\mu_N}\left(1-\frac{1}{N}\right)\right\}} \prod_{j=1,j\neq i}^{N} 1_{\left\{\frac{h_j}{\lambda_N+\mu_N g_j} \leq F^{-1}_{\lambda_N,\mu_N}\left(1-\frac{1}{N}\right)\right\}}, \tag{25}$$

which implies that $\lim_{N\to\infty} \mathsf{E}\left[1_{\{A_N\}}\right] = \frac{1}{e}$. This shows the logarithmic effect of P_{ave} on the secondary network throughput. Using Lemma 2, we have $\frac{\log\left(X^\star_N\left(1,\frac{\mu_N}{\lambda_N}\right)\right)}{\log\log(N)} \xrightarrow{i.p.} \frac{1}{n_h}$ since λ_N can not be arbitrarily small and $\mu_N \leq \frac{1}{Q_{\text{ave}}}$. Also, we have $1_{\{A_N\}} \xrightarrow{i.d.} Bern\left(\frac{1}{e}\right)$. Applying Slutsky Theorem, we have $\frac{\log\left(X^\star_N\left(1,\frac{\mu_N}{\lambda_N}\right)\right)}{\log\log(N)} 1_{\{A_N\}} \xrightarrow{i.d.} \frac{1}{n_h} Bern\left(\frac{1}{e}\right)$. Let $\hat{X}_N\left(1,\frac{\mu_N}{\lambda_N}\right) = \frac{\log\left(X^\star_N\left(1,\frac{\mu_N}{\lambda_N}\right)\right)}{\log\log(N)} 1_{\{A_N\}}$. Since convergence in distribution does not always imply convergence in mean [20], we need to show that the collection of random variables $\left\{\hat{X}_N\left(1,\frac{\mu_N}{\lambda_N}\right)\right\}_{N=1}^{\infty}$ is uniformly integrable, *i.e.*, $\lim_{C'\to\infty}\sup_N \mathsf{E}\left[\left|\hat{X}_N\left(1,\frac{\mu_N}{\lambda_N}\right)\right| 1_{\left\{\left|\hat{X}_N\left(1,\frac{\mu_N}{\lambda_N}\right)\right|\geq C'\right\}}\right] = 0$. For N large enough, we have $\frac{\log\left(X^\star_N\left(1,\frac{\mu_N}{\lambda_N}\right)\right)}{\log\log(N)} 1_{\{A_N\}} \leq \frac{\log\left(X^\star_N\left(1,\frac{\mu_N}{\lambda_N}\right)\right)}{\log\log(N)} 1_{\left\{X^\star_N\left(1,\frac{\mu_N}{\lambda_N}\right)\geq 1\right\}}$. Also, using similar technique to the proof of Lemma 3 in [11], it is easy to show that $\left\{\frac{\log\left(X^\star_N\left(1,\frac{\mu_N}{\lambda_N}\right)\right)}{\log\log(N)} 1_{\left\{X^\star_N\left(1,\frac{\mu_N}{\lambda_N}\right)\geq 1\right\}}\right\}_{N=1}^{\infty}$ is uniformly integrable which implies that $\left\{\frac{\log\left(X^\star_N\left(1,\frac{\mu_N}{\lambda_N}\right)\right)}{\log\log(N)} 1_{\{A_N\}}\right\}_{N=1}^{\infty}$ is uniformly integrable.

Appendix B
Proof of Theorem 2

To prove Theorem 2, first, we show that $\liminf_{N\to\infty} Np^\star_N \geq 1$ by contradiction. Assume that $\liminf_{N\to\infty} Np^\star_N = a$ where $0 \leq a < 1$. Then, we can find a subsequence of N, N_j such that $\lim_{N_j\to\infty} N_j p^\star_{N_j} = a$. Consider the case of $a \neq 0$. In this case, $\lim_{N_j\to\infty} \mathsf{Pr}\left(A_{N_j}\right) = \frac{a}{\mathrm{e}^a}$. Using similar technique to the proof of Theorem 1, we can show that $\lim_{N_j\to\infty} \frac{R_{\text{DTPIL}}\left(p^\star_{N_j}, N_j\right)}{\log\log(N_j)} = \frac{a}{\mathrm{e}^a n_h} < \frac{1}{\mathrm{e} n_h}$. Note that we can obtain better throughput scaling by the choice of $p_{N_j} = \frac{1}{N_j}$ which contradicts with the fact that $p^\star_N$ is the optimal transmission probability. Similarly for the case of $a = 0$, we can show that $\lim_{N_j\to\infty} \frac{R_{\text{DTPIL}}\left(p^\star_{N_j}, N_j\right)}{\log\log(N_j)} = 0$ (This result can be intuitively obtained by setting a to zero in the previous discussions). Hence, $\liminf_{N\to\infty} Np^\star_N \geq 1$. Similarly, we can show that $\limsup_{N\to\infty} Np^\star_N \leq 1$ which completes the proof.

Appendix C
Proof of Theorem 3

Note that $R_{\text{DTPIL}}\left(p^\star_N, N\right) \geq R_{\text{DTPIL}}\left(\frac{1}{N}, N\right)$. Hence, we have $\liminf_{N\to\infty} \frac{R_{\text{DTPIL}}(p^\star_N, N)}{\log\log(N)} \geq \frac{1}{\mathrm{e} n_h}$. To show the other side, let $\tilde{X}_N = \frac{\log(X^\star_N(\lambda_N,\mu_N))}{\log\log(N)}$. For all $\epsilon > 0$, we have

$$\begin{aligned}
\frac{R_{\text{DTPIL}}\left(p^\star_N, N\right)}{\log\log\left(N\right)} &= \mathsf{E}\left[\tilde{X}_N 1_{\{A_N\}} 1_{\left\{\left|\tilde{X}_N - \frac{1}{n_h} > \epsilon\right|\right\}}\right] + \mathsf{E}\left[\tilde{X}_N 1_{\{A_N\}} 1_{\left\{\left|\tilde{X}_N - \frac{1}{n_h}\right| \leq \epsilon\right\}}\right] \\
&\leq \mathsf{E}\left[\tilde{X}_N 1_{\{X^\star_N(\lambda_N,\mu_N)\geq 1\}} 1_{\left\{\left|\tilde{X}_N - \frac{1}{n_h} > \epsilon\right|\right\}}\right] + \left(\frac{1}{n_h} + \epsilon\right) \mathsf{E}\left[1_{\{A_N\}}\right].
\end{aligned} \tag{26}$$

Note that $\tilde{X}_N 1_{\{X_N^\star(\lambda_N,\mu_N)\geq 1\}} \xrightarrow{i.p.} \frac{1}{n_h}$ and $1_{\left\{\left|\tilde{X}_N-\frac{1}{n_h}>\epsilon\right|\right\}} \xrightarrow{i.p.} 0$, hence $\tilde{X}_N 1_{\{X_N^\star(\lambda_N,\mu_N)\geq 1\}} 1_{\left\{\left|\tilde{X}_N-\frac{1}{n_h}>\epsilon\right|\right\}} \xrightarrow{i.p.} 0$. Similar to the proof of Theorem 1, it is easy to show that the collection of random variables

$$\left\{\tilde{X}_N 1_{\{X_N^\star(\lambda_N,\mu_N)\geq 1\}} 1_{\left\{\left|\tilde{X}_N-\frac{1}{n_h}>\epsilon\right|\right\}}\right\}_{N=1}^{\infty},$$

is uniformly integrable which implies $\lim_{N\to\infty} \mathsf{E}\left[\tilde{X}_N 1_{\{X_N^\star(\lambda_N,\mu_N)\geq 1\}} 1_{\left\{\left|\tilde{X}_N-\frac{1}{n_h}>\epsilon\right|\right\}}\right] = 0$.

For N large enough, $\mathsf{E}\left[1_{\{A_N\}}\right]$ can be written as

$$\begin{aligned} \mathsf{E}\left[1_{\{A_N\}}\right] &= \mathsf{Pr}\left(A_N\right) \\ &= N p_N^\star \left(1-p_N^\star\right)^{N-1} \\ &\overset{a}{\leq} \left(1-\frac{1}{N}\right)^{N-1}, \end{aligned} \tag{27}$$

where (a) follows from the fact that $N p_N^\star \left(1-p_N^\star\right)^{N-1}$ is maximized at $p_N^\star = \frac{1}{N}$. Hence,

$$\limsup_{N\to\infty} \frac{R_{\mathrm{DTPIL}}\left(p_N^\star, N\right)}{\log\log\left(N\right)} \leq \frac{1}{e n_h}, \tag{28}$$

which completes the proof.

APPENDIX D
THROUGHPUT SCALING IN DIL NETWORKS

In the next Lemma, we study asymptotic behavior of μ_N in DIL networks. This Lemma will be helpful to study the effect of average total interference power, Q_{ave}, on the secondary network throughput in DIL networks.

Lemma 4: Let μ_N be the power control parameter in the DIL network. Then, $\lim_{N\to\infty} \mu_N = \frac{1}{Q_{\mathrm{ave}}}$.

Proof: First, note that $\mu_N \leq \frac{1}{Q_{\mathrm{ave}}}$. Thus, for N large enough, we have

$$\begin{aligned} \mu_N &= \frac{1}{Q_{\mathrm{ave}}} \mathsf{E}\left[\sum_{i=1}^{N}\left(1-\frac{\mu_N g_i}{h_i}\right) 1_{\left\{\frac{h_i}{g_i}>\max\left(F^{-1}\left(1-\frac{1}{N}\right),\mu_N\right)\right\}}\right] \\ &= \frac{1}{Q_{\mathrm{ave}}} - \mu_N \sum_{i=1}^{N} \mathsf{E}\left[\frac{g_i}{h_i} 1_{\left\{\frac{h_i}{g_i}>F^{-1}\left(1-\frac{1}{N}\right)\right\}}\right]. \end{aligned} \tag{29}$$

Since μ_N can not be arbitrary large, the second term in (29) converges to zero as N tends to infinity which completes the proof. ■

The sum-rate in DIL networks can be written as

$$R_{\mathrm{DIL}}\left(\frac{1}{N}, N\right) = \log\left(\frac{1}{\mu_N}\right) \mathsf{E}\left[1_{\{B_N\}}\right] + \mathsf{E}\left[\log\left(Y_N^\star\right) 1_{\{B_N\}}\right]. \tag{30}$$

For N large enough, we have

$$1_{\{B_N\}} = \sum_{i=1}^{N} 1_{\left\{\frac{h_i}{g_i}>F^{-1}\left(1-\frac{1}{N}\right)\right\}} \prod_{j=1,j\neq i}^{N} 1_{\left\{\frac{h_j}{g_j}\leq F^{-1}\left(1-\frac{1}{N}\right)\right\}}, \tag{31}$$

Hence, $\lim_{N\to\infty} \mathsf{E}\left[1_{\{B_N\}}\right] = \frac{1}{e}$. Thus, the first term in (30) converges to $\frac{1}{e}\log\left(Q_{\text{ave}}\right)$ as N tends to infinity which shows the logarithmic effect of average interference power constraint, Q_{ave}, on the secondary network sum-rate under DIL networks. Note that $\frac{\log(Y_N^\star)}{\log(N)} \xrightarrow{i.p.} \frac{1}{\gamma_g}$ (see [11] Lemma 8 in). Also, $1_{\{B_N\}} \xrightarrow{i.d.} Bern\left(\frac{1}{e}\right)$. Using Slutsky theorem, we have $\frac{\log(Y_N^\star)}{\log(N)}1_{\{B_N\}} \xrightarrow{i.d.} Bern\left(\frac{1}{e}\right)$. Since convergence in distribution does not alway imply convergence in mean, we need to show that the sequence of random variables $\left\{\frac{\log(Y_N^\star)}{\log(N)}1_{\{B_N\}}\right\}_{N=1}^{\infty}$ is uniformly integrable. For N large enough, we have $\frac{\log(Y_N^\star)}{\log(N)}1_{\{B_N\}} \leq \frac{\log(Y_N^\star)}{\log(N)}1_{\{Y_N^\star\geq 1\}}$. Based on Lemma 8 in, [11] $\left\{\frac{\log(Y_N^\star)}{\log(N)}1_{\{Y_N^\star\geq 1\}}\right\}_{N=1}^{\infty}$ is uniformly integrable which implies that $\left\{\frac{\log(Y_N^\star)}{\log(N)}1_{\{B_N\}}\right\}_{N=1}^{\infty}$ is uniformly integrable. Hence, we have $\lim_{N\to\infty} \mathsf{E}\left[\frac{\log(Y_N^\star)}{\log(N)}1_{\{B_N\}}\right] = \frac{1}{e\gamma_g}$.

APPENDIX E
PROOF OF THEOREM 5

First, we show that $\liminf_{N\to\infty} Np_N^\star \geq 1$ by contradiction. Assume that $\liminf_{N\to\infty} Np_N^\star = a$ where $0 \leq a < 1$. Find a subsequence of N, N_j, such that $\lim_{N_j\to\infty} N_j p_{N_j}^\star = a$. Consider the case of $a \neq 0$. In this case, $\lim_{N_j\to\infty} \mathsf{Pr}\left(B_{N_j}\right) = \frac{a}{\mathrm{e}^a}$ which implies that the portion of time that just the SU with the maximum of $\frac{h_i}{g_i}$ transmits is close to $\frac{a}{\mathrm{e}^a}$ as N becomes large. Following similar steps to the proof of Theorem 4, we can show that $\lim_{N_j\to\infty} \frac{R_{\text{DIL}}\left(p_{N_j}^\star, N_j\right)}{\log(N_j)} = \frac{a}{\mathrm{e}^a\gamma_g}$ which contradicts with the optimality of $p_N^\star$ since we can asymptotically obtain better scaling result by $p_{N_j} = \frac{1}{N_j}$. Similarly, for the case of $a = 0$, we can show that $\lim_{N_j\to\infty} \frac{R_{\text{DIL}}\left(p_{N_j}^\star, N_j\right)}{\log(N_j)} = 0$ which contradicts with the fact that $p_N^\star$ is optimal transmission probability. Hence, $\liminf_{N\to\infty} Np_N^\star \geq 1$. Similarly, we can show that $\limsup_{N\to\infty} Np_N^\star \leq 1$ which completes the proof.

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